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ULTRASONIC MEASUREMENT OF THE VELOCITY OF SOUND IN DISTILLED AND SEA WATER (U)

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ULTRASONIC MEASUREMENT OF THE VELOCITY OF SOUND IN DISTILLED AND SEA WATER

Prepared by:

W. D. Wilson

ABSTRACT: An instrument has been designed and constructed for the measurement of sound speed in liquids as a function of temperature and pressure. This instrument utilizes the ultrasonic pulse technique to determine the time required for a pulse to traverse the distance between two crystal transducers fixed at each end of a cylinder containing the test liquid. A description of the instrument and the accessory equipment required for its use is given in this report. The results obtained in distilled water and in sea water are tabulated and compared with the results obtained in other investigations. It is estimated that the precision of the measurements in water is 1 part in 7,500.

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This report describes the design, construction, theory and use of a velocimeter for the precision measurement of the speed of sound in liquids and it presents original data on the speed of sound in distilled water and in samples of sea water. The present work was supported by Foundational Research funds under the "Liquid State" task, FR-54. The report will be of interest principally to other physicists interested in precision measurements of the speed of sound and in basic research on the liquid state.

J. A. QUENSE', Acting
Captain, USN
Commander

S. J. RAFF
By direction

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ULTRASONIC MEASUREMENT OF THE VELOCITY OF SOUND IN DISTILLED AND SEA WATER

INTRODUCTION

1. A review of the literature concerned with the thermodynamic properties of liquids indicates that very little work has been done to determine the influence of pressure on the parameters involved. The work by P. W. Bridgeman[1] represents an extensive study in liquids under high pressure and it is often quoted. There are many studies in liquid state physics, however, which require a detailed investigation in some intermediate pressure range. The ultrasonic velocimeter at the Naval Ordnance Laboratory was constructed primarily to measure the speed of sound in water over a pressure range 14.7 psi $\leq P \leq$ 14,000 psi and a temperature range of $-4.0^{\circ}C \leq T < 100^{\circ}C$. The instrument is quite versatile, however, and may be used for other liquids and for pressures up to 100,000 psi.

2. Of the numerous instruments that have been designed to measure the speed of sound in liquids three have been constructed to operate at high pressures[2,3,4]. The instrument with the greatest precision, however, is an instrument designed by M. Greenspan and C. Tschiegg at the National Bureau of Standards[5] for measurements of sound speed at atmospheric pressure only. This instrument uses crystal transducers which are fixed in position. This arrangement is convenient for measurements under pressure. The basic design of this instrument has been modified to accommodate the transmission of pressure to the test sample and has proved to be quite satisfactory for the purpose intended in the NOL velocimeter.

3. A knowledge of the speed of sound in liquids, when combined with accurate specific volume data enables one to compute all the thermodynamic quantities required[6]. Unfortunately, there is very little information available on the effect of pressure on the specific volume of liquids. The results obtained from the velocimeter are therefore limited at the present time to the single purpose of measuring the speed of sound in liquids as a function of temperature and pressure. The present report is concerned with an adequate description of the velocimeter, its associated instrumentation, and with the results obtained in water.

THEORY AND DESIGN OF THE VELOCIMETER

4. The actual measurement of sound speed is accomplished in a cylindrical housing which is 4.997934 ±0.000005" long and has a ½" bore. Each end of the cylinder is terminated by a 5 mc, gold plated, quartz crystal for the transmission and reception of sound. The test liquid is placed in this chamber for the measurement of sound speeds. The bore diameter of the cylinder is approximately 12.5 times the wave length of sound in the crystal. The transmitting crystal is driven by a series of pulses, each of 0.05 μsec duration, with a repetition frequency which depends on the length of the acoustic path. The acoustic path is taken, to a good approximation, to be the physical distance between the transmitting and receiving crystals. When a pulse is applied to the transmitting crystal, a 5 mc wave train is transmitted through the liquid. This wave train is reflected back upon itself by the receiving crystal and is returned to the transmitting crystal. As it arrives at the transmitting crystal, a second pulse is initiated. The initial wave train and the wave train resulting from the second pulse then travel through the liquid together. The superimposed waves are detected by the receiving crystal and displayed on an oscilloscope. The repetition frequency of the pulse is varied until the first half cycles of the wave trains coincide. The pulse repetition frequency indicates the time required for the sound waves to travel twice the length of the acoustic path. The speed of sound in the liquid sample is computed from this time and from the known distance between crystals.

5. The velocimeter was constructed with great care. The ends of the cylinder were accurately machined plane parallel to each other and perpendicular to the axis. The back of the crystals are first attached to Mycalex insulators with vacuum grease and this assembly is then forced against the ends of the cylinder by compression springs. The crystals used were gold plated to provide electrical contacts. Since sound speeds were to be obtained under pressure, neoprene O-rings were employed between the crystals and the cylinder to provide a liquid seal. The depths of the "O"-ring grooves were designed to allow the O-rings to compress under the force of the springs. This allowed the crystals to make uniform contact with the ends of the cylinder when the end caps were secured in position. This construction permitted easy assembly and disassembly of the velocimeter. Fig. 1 shows the construction of the velocimeter.

SCHEMATIC OF VELOCIMETER

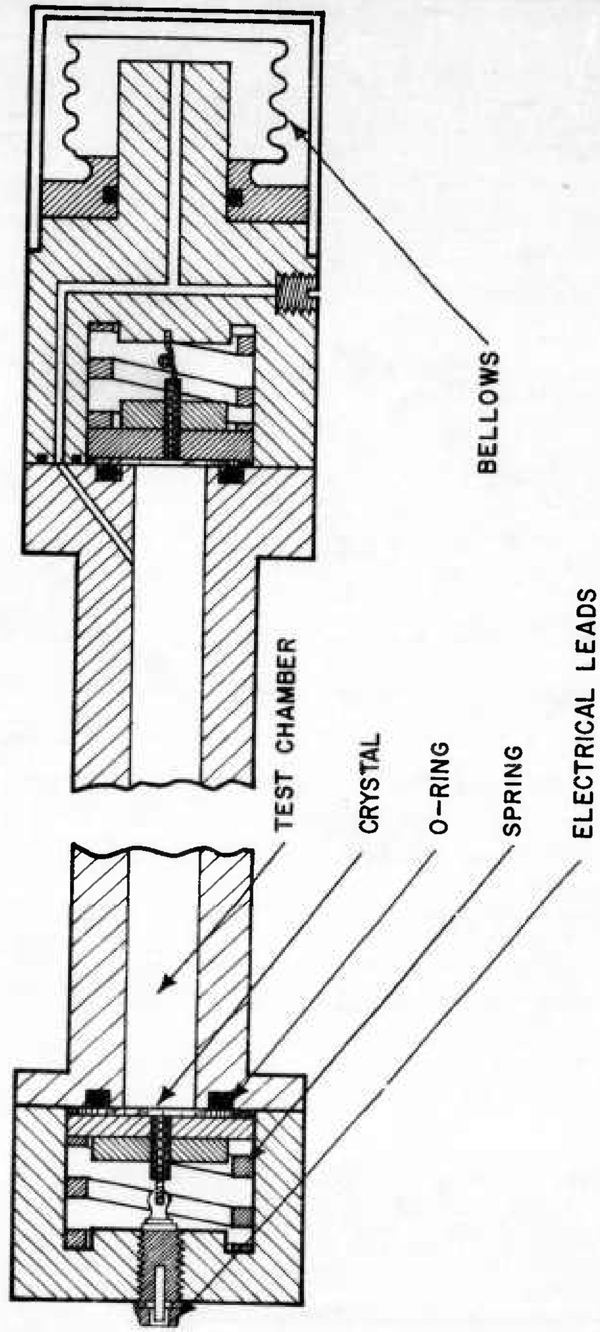


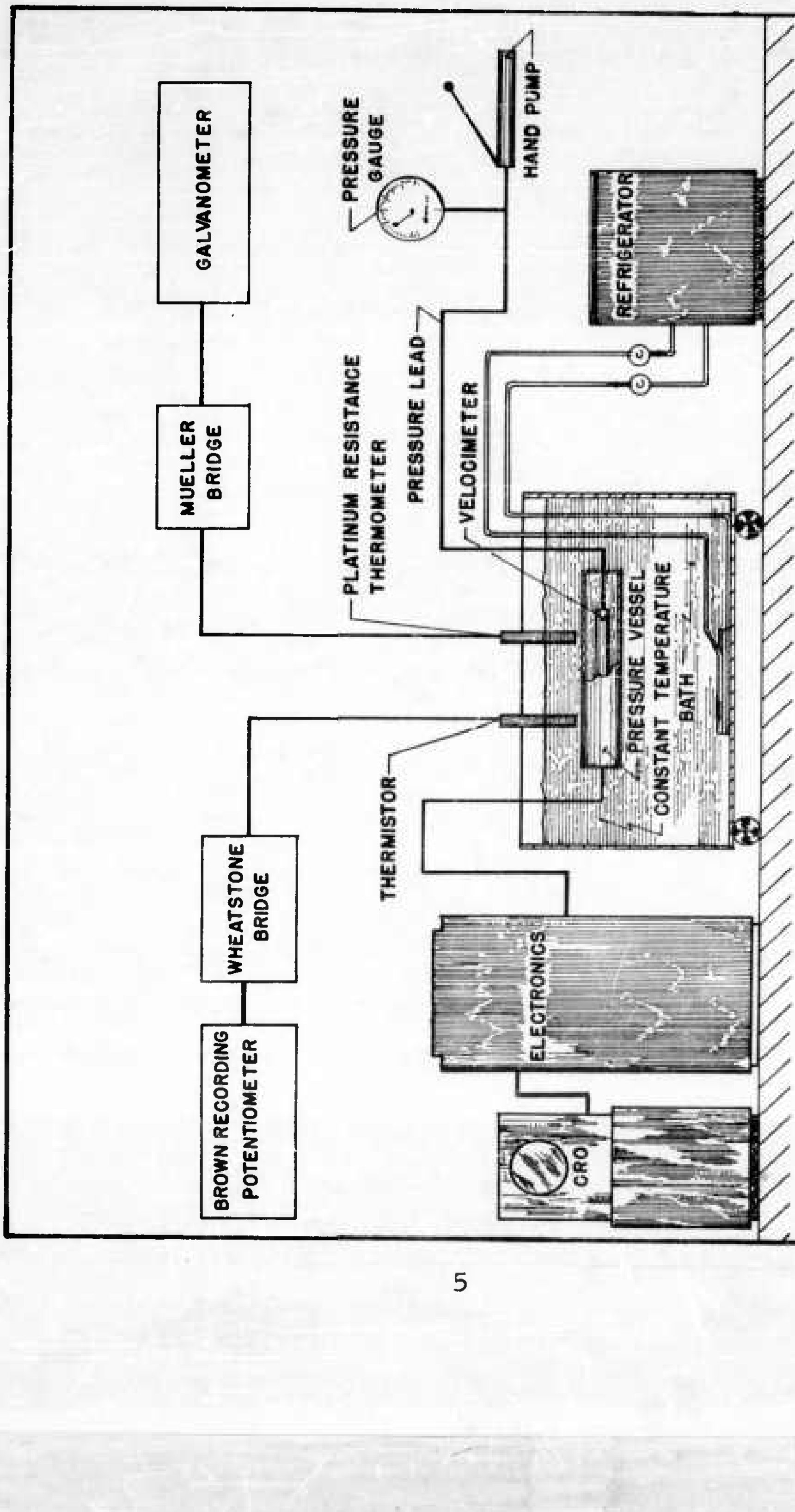
FIG. 1

6. The length of the test chamber is sufficient to prevent overlapping of the wave train. The pulses are consequently separated in time to prevent the first cycle of a wave train from being affected by the later portions of the previous wave train when coincidence is being established. It is believed[7] that wave-crystal interaction during reflection affects the phase of that portion of the wave which arrives after the first cycle. This is particularly true if the two crystals are not matched in frequency. In the NOL instrument, the time required for sound to traverse the length of the test chamber is approximately five times the duration of the wave train (in water).

7. The transmission of pressure to the test liquid is accomplished by attaching a bellows to one end cap of the velocimeter. The bellows is filled with the test liquid and communicates hydrostatically with the liquid in the test chamber. The bellows is made of stainless steel and a force is required to compress it. This force is a function of the pressure applied and the compressibility of the liquid sample. A soft bellows is used to obtain a minimum pressure drop across the bellows. In the NOL instrument, the pressure in a distilled water sample is 0.89 psi lower than the pressure indicated outside the velocimeter when the hydrostatic pressure field outside is 15,000 psi.

ASSOCIATED INSTRUMENTATION

8. A schematic of the instrumentation required for monitoring and controlling the physical environment of the velocimeter is shown in Fig. 2. The velocimeter is placed in its pressure vessel and this assembly is lowered into a 110 gallon constant temperature bath. The bath liquid is a water-alcohol mixture which is continuously agitated by three circulating pumps. The temperature of the bath is controlled by a mercury thermoregulator. An external refrigeration unit is used to obtain the low temperature. The thermistor shown in Fig. 2 is used initially as a probe to sense temperature gradients in the bath. Such gradients were eliminated by installing three pumps to circulate the bath liquid. The thermistor is used also to detect fluctuations in bath temperature caused by the regulating system. By controlling the rate of heating and cooling of the bath, these fluctuations can be reduced to 0.0005°C peak to peak at all temperatures between -3°C and +90°C. Finally, the absolute temperature of the bath is measured with a platinum resistance thermometer.



INSTRUMENTATION FOR THE MEASUREMENT OF THE VELOCITY OF SOUND IN LIQUIDS

FIG. 2

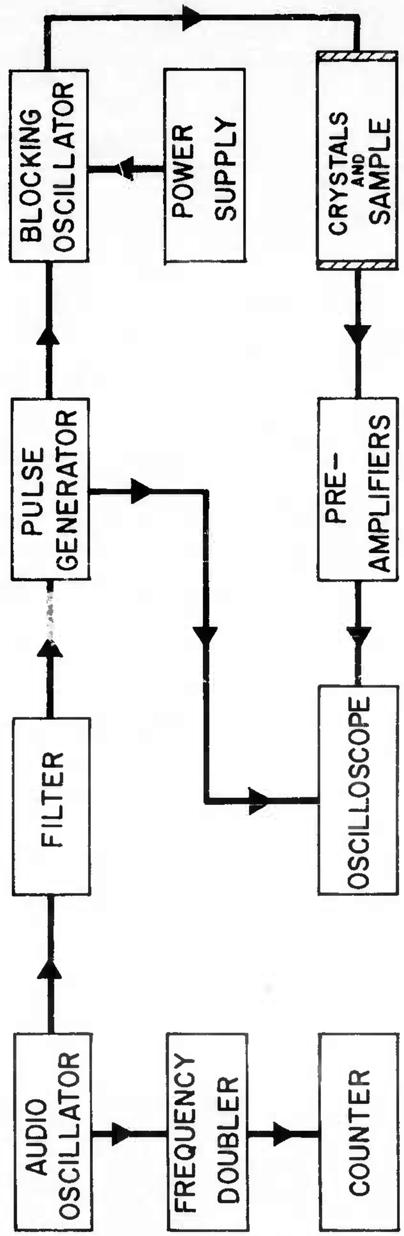
9. Since a change in pressure of 2000 psi can change the temperature of the test liquid nearly 1.0°C, it was necessary to allow the test liquid and the bath to come to thermal equilibrium. Equilibrium was determined by monitoring the speed of sound until a constant value was observed. Experience with the system indicated that it requires nearly one hour for the temperature inside and outside the pressure vessel to stabilize after a pressure change of 2000 psi.

10. During the measurements of sound speed in distilled water the pressure was determined by a deadweight tester, a sensitive Heise pressure gauge, and a manganin resistance cell. Due to difficulties encountered in the use of the manganin cell, the deadweight tester was relied upon for the pressures recorded. Prior to the measurements of sound speed in sea water the difficulties experienced with the manganin cell were overcome and it was used to determine the actual pressure during these measurements. The manganin cell was calibrated against a high precision deadweight tester. The resistance change of the manganin cell was a linear function of the pressure. It is estimated that the pressures recorded for the distilled water results are accurate to within ±7 psia and that those recorded for the sea water results are accurate to within ±2 psia. All pressures recorded in this report are absolute.

11. The electronic instrumentation shown in Fig. 2 is expanded in the block diagram of Fig. 3. With the exception of the pulse generator and the blocking oscillator, all components are commercially available. The pulse generator and the blocking oscillator circuits (shown in Fig. 4) are standard circuits and do not require detailed explanation. It should be noted, however, that the duration of the pulse should be short compared to the natural period of the crystals. The duration of the pulse in the NOL velocimeter is approximately $\frac{1}{4}$ the crystal period with an open circuit amplitude of 80 volts. The highest efficiency is obtained when the pulse duration is slightly greater than $\frac{1}{2}$ the natural period of the crystal. The output of the blocking oscillator is rectified so that only the positive half cycle of the pulse is applied across the transducer.

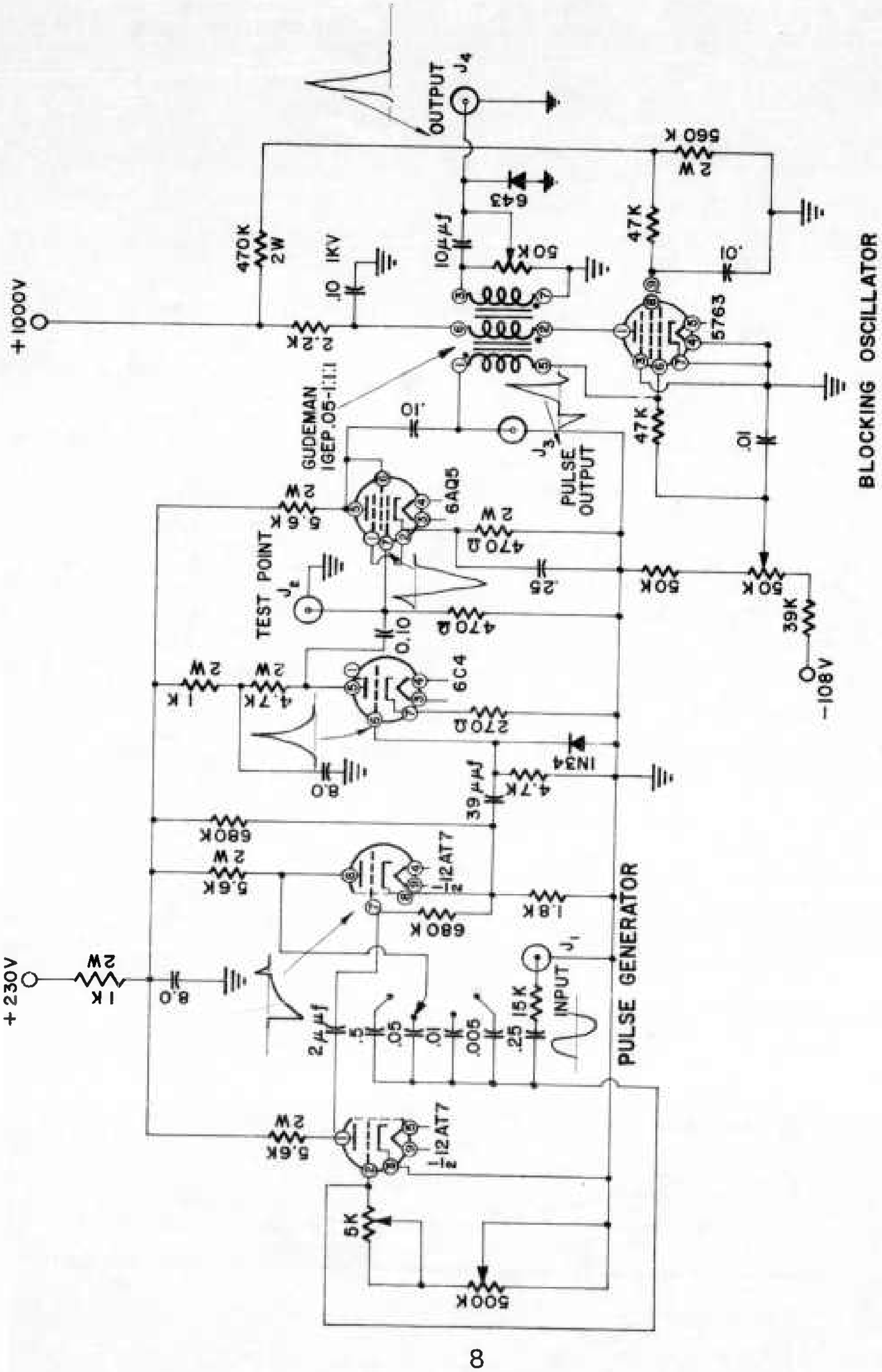
METHOD OF MEASUREMENT

12. The precision with which the temperature, pressure, and frequency must be measured depends upon the effect each has on the speed of sound and upon the accuracy required. The change in sound speed per degree change in temperature in the neighborhood of 0°C is about 5.0 m/sec/deg for water. To obtain an



BLOCK DIAGRAM OF ELECTRONICS

FIG. 3



PULSE GENERATING CIRCUITRY

FIG. 4

accuracy of 1 part in 30,000 for the speed of sound in water, the temperature must be known to the nearest 0.01°C; actually, the temperature was measured to the nearest 0.001°C and rounded to the nearest 0.01°C after all calculations were completed. In regards to pressure, it is found that a unit change in pressure will change the speed of sound in water by 0.011 m/sec/psi in the neighborhood of atmospheric pressure. This coefficient does not vary appreciably in the pressure range considered so it may be estimated that a change of 4 psi will not affect the speed of sound measurements beyond the precision required. It may be recalled from above that the pressure can be measured to within ±2 psi with the manganin resistance cell. The change in sound speed per unit change in frequency is 0.127 m/sec/cps. A maximum change of 1.2 cps may be tolerated if the above accuracy is to be obtained. Ten readings of frequency were obtained for the computation of each sound speed data point. These measurements were required to have a standard deviation from their mean of 0.2 cps; if this condition did not occur the ten readings were discarded and a new set obtained. This allowed frequency variation included a ±0.1 cps variation due to the accuracy of the counter and the variations in frequency resulting from the ability of the operator to set coincidence of the sound waves.

13. It should be noted that the measurement of each parameter was individually based on a desired precision of 1 part in 30,000. Since the errors associated with each measurement add to give the final error in the sound speed measurements care must be taken that their sum does not exceed the precision desired in the final sound speed tables.

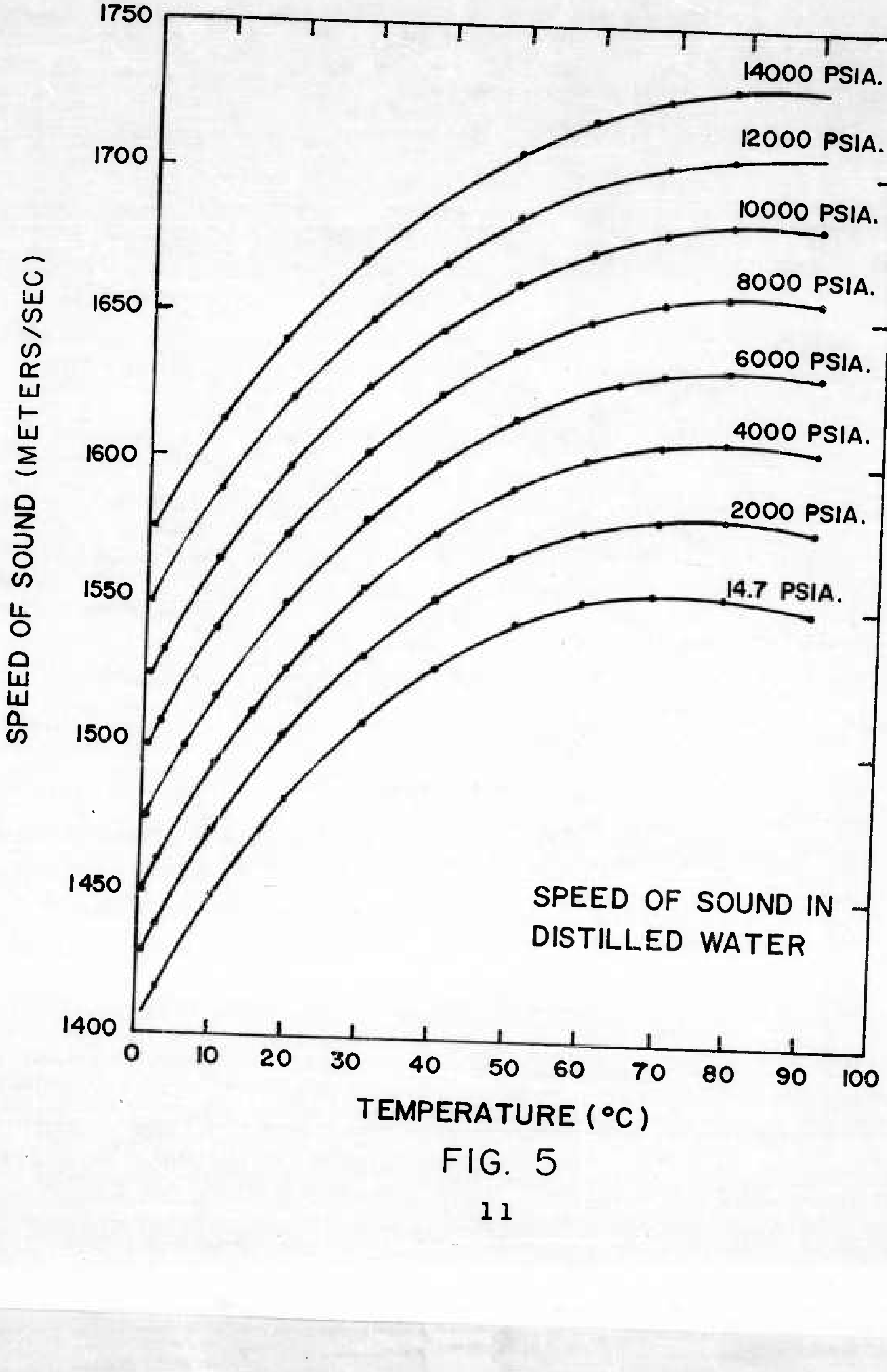
14. The actual measurements of sound speed were obtained by first adjusting the temperature and then, for this temperature, to vary the pressure over the range required. Approximately one hour was allowed to elapse after each pressure change before the repetition frequency was measured. This is to assure that the temperature and the pressure in the test liquid has come to equilibrium with the temperature of the bath and the pressure in the manganin cell. The frequency itself was used to determine when the equilibrium conditions were established. When this condition was obtained ten measurements of the repetition frequency were recorded. The average of these ten readings were then used to compute the speed of sound. This computation also includes a correction for the change in length of the velocimeter due to temperature and pressure changes.

EXPERIMENTAL RESULTS IN WATER

15. Values for the speed of sound in distilled water were obtained for eleven temperatures and eight pressures in the temperature range $0 < T \; 100^{o}C$ and the pressure range 14.7 psia $\leq P \leq$ 14,000 psia. These data points are plotted in Fig. 5 with temperature as abscissa and in Fig. 6 with pressure as abscissa. The experimental results are also tabulated in Table I. In sea water, 581 measurements of sound speed were made in the range $33‰ < S < 37‰$, $-3^{o}C < T < 30^{o}C$, and $14.7 \leq P \leq 14{,}000$ psi, where S, T, and P refer respectively to salinity, temperature, and pressure. These results are tabulated in Tables II through VI and in Figs. 7 through 11. The sea water samples were furnished by the Navy Hydrographic Office and are actual sea water taken from the Bermuda-Key West area of the Atlantic Ocean. Empirical equations have been fitted to the distilled water data[8] and to the sea water data[9]; these equations and their associated tables are not given here since they have been reported in the references given.

DISCUSSION OF RESULTS IN WATER

16. It may be seen in Fig. 5 that the maximum sound speed in distilled water shifts toward higher temperatures as higher pressures are considered. This behavior agrees with the results obtained by A. Smith and A. Lawson[3] and by T. Litovitz and E. Carnevale[4]. It is contrary to the results of G. Holton[2]. From Table I and Tables II through VI it is seen that the measured sound speeds in water at atmospheric pressure exceed the predicted values of S. Kuwahara[10] and D. J. Matthews[11] by about 3 m/sec. This difference between the predicted and the measured sound speeds was observed earlier in distilled water and in sea water by V. Del Grosso[12] and it is further substantiated here. It is apparent that the tables of sound speed in current use, the Kuwahara and the Matthews tables, are in error at atmospheric pressure. At pressures greater than atmospheric pressure further differences are observed. It is seen in Fig. 6 that the curves of sound speed vs pressure in distilled water are concave upward for temperatures less than $20^{o}C$ and concave downward for temperatures greater than $20^{o}C$ over the pressure range of 14.7 to 14,000 psia. This is also approximately true for sea water. The Kuwahara and the Matthews tables, when plotted against pressure, give curves which are concave downward for all temperatures. This contrast is shown clearly in Fig. 12 for sea water of salinity 33.05 ‰ .



1750
1700
1650
1600
1550
1500
1450
1400
SPEED OF SOUND (METERS/SEC)
14000 PSIA.
12000 PSIA.
10000 PSIA.
8000 PSIA.
6000 PSIA.
4000 PSIA.
2000 PSIA.
14.7 PSIA.
SPEED OF SOUND IN
DISTILLED WATER
0
10
20
30
40
50
60
70
80
90
100
TEMPERATURE (°C)

FIG. 5

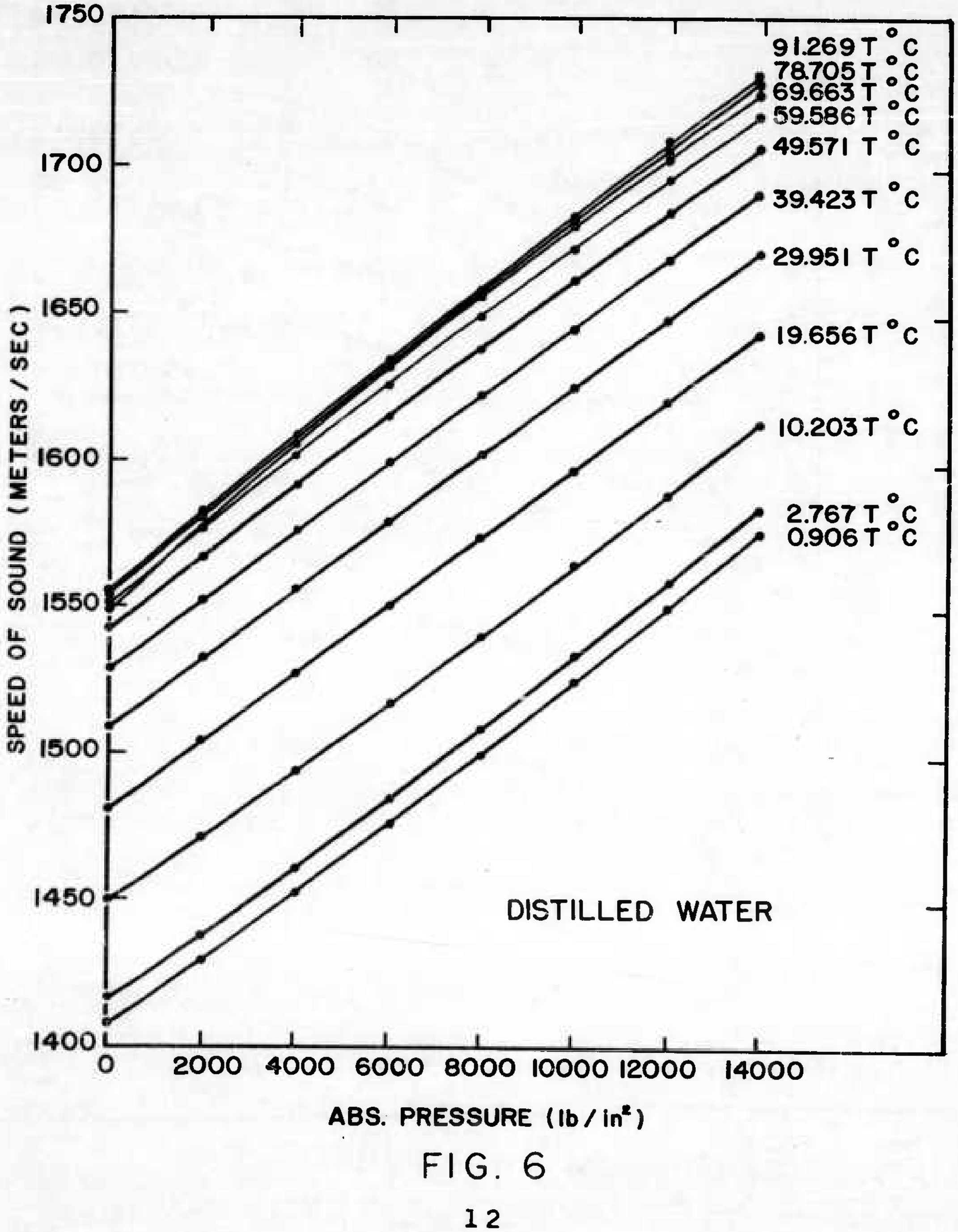

91.269 T °C
78.705 T °C
69.663 T °C
59.586 T °C
49.571 T °C
39.423 T °C
29.951 T °C
19.656 T °C
10.203 T °C
2.767 T °C
0.906 T °C
DISTILLED WATER
SPEED OF SOUND (METERS / SEC)
1750
1700
1650
1600
1550
1500
1450
1400
0
2000
4000
6000
8000
10000
12000
14000
ABS. PRESSURE (lb / in²)

FIG. 6

TABLE I

MEASURED SPEED OF SOUND IN DISTILLED WATER

T°C / P lbs/in^2	0.906°	2.767°	10.203°	19.656°	29.951°	39.423°	49.571°	59.586°	69.663°	78.705°	91.269°
14.7	1407.41	1416.35	1449.05	1481.63	1509.37	1528.36	1542.60	1551.01	1555.02	1554.90	1549.80
2,000	1429.25	1438.06	1471.46	1504.34	1532.67	1552.03	1567.11	1576.21	1581.13	1581.83	1578.03
4,000	1451.66	1460.83	1494.17	1527.38	1555.87	1575.72	1591.22	1600.96	1606.58	1607.97	1605.31
6,000	1475.37	1484.42	1517.28	1550.49	1579.04	1599.16	1614.92	1625.21	1631.44	1633.35	1631.75
8,000	1499.72	1508.50	1540.99	1573.79	1602.12	1622.17	1638.22	1648.91	1655.69	1657.95	1657.36
10,000	1524.61	1533.29	1564.78	1596.90	1625.06	1645.14	1661.28	1672.30	1679.34	1682.13	1682.18
12,000	1549.93	1558.09	1588.75	1620.25	1647.88	1667.72	1684.00	1695.13	1702.55	1705.65	1706.39
14,000	1575.22	1583.15	1612.66	1643.41	1670.58	1690.41	1706.51	1717.68	1725.28	1728.69	1730.02

TABLE II

MEASURED SPEED OF SOUND IN SEA WATER, s = 33.08 ‰

T°C / P lbs/in²	-2.166°	-0.953°	0.003°	0.906°	2.153°	3.094°	4.050°	4.947°	6.035°	10.093°	14.880°	19.994°	24.946°	29.888°
14.7	1436.16	1442.00	1446.60	1450.80	1456.30	1460.34	1464.22	1467.80	1472.52	1487.61	1504.18	1519.43	1532.32	1543.43
2,000	1458.33	1464.34	1468.97	1473.19	1478.77	1482.87	1486.71	1490.31	1494.97	1510.06	1526.85	1542.17	1555.35	1566.51
4,000	1481.43	1487.49	1491.85	1496.16	1501.85	1505.87	1509.68	1513.14	1517.84	1533.03	1549.62	1564.99	1578.12	1589.57
6,000	1505.26	1511.03	1515.42	1519.72	1525.24	1529.22	1533.10	1536.42	1541.20	1556.05	1572.60	1587.97	1600.93	1612.28
8,000	1529.77	1535.28	1539.62	1543.70	1549.06	1553.05	1556.75	1560.13	1564.84	1579.46	1595.67	1610.91	1623.72	1635.04
10,000	1554.65	1559.97	1564.26	1568.05	1573.24	1577.12	1580.84	1583.96	1588.55	1602.84	1618.65	1633.91	1646.42	1657.59
12,000	1579.89	1584.89	1589.17	1592.71	1597.68	1601.37	1604.97	1607.97	1612.46	1626.35	1641.78	1656.67	1669.16	1679.97
14,000	1605.15	1609.87	1613.94	1617.12	1622.00	1625.58	1629.08	1631.96	1636.44	1649.91	1664.84	1679.47	1691.61	1702.29

TABLE III

MEASURED SPEED OF SOUND IN SEA WATER, s = 33.95 ‰

T°C / P lbs/in²	-3.093°	-1.961°	-0.906°	0.090°	1.007°	2.100°	3.083°	3.906°	5.015°	6.131°	9.855°	14.952°	19.876°	25.162°	29.937°
14.7	---	1438.91	1444.01	1448.47	1452.71	1457.50	1461.77	1465.27	1469.87	1474.49	1488.58	1505.70	1520.48	1534.00	1544.73
2,000	1455.64	1461.16	1466.55	1470.87	1475.12	1480.13	1484.19	1487.63	1492.30	1496.70	1511.02	1528.37	1543.21	1556.92	1567.79
4,000	1478.90	1484.49	1489.64	1494.11	1498.10	1503.08	1507.24	1510.74	1515.24	1519.79	1533.73	1551.20	1565.96	1579.87	1590.74
6,000	1503.06	1508.46	1513.46	1517.88	1521.83	1526.72	1530.85	1534.25	1538.66	1543.25	1557.03	1574.11	1588.86	1602.68	1613.76
8,000	1527.56	1532.94	1537.80	1542.10	1545.95	1550.85	1554.74	1558.11	1562.47	1566.84	1580.39	1597.25	1611.83	1625.45	1636.46
10,000	1552.43	1557.74	1562.49	1566.67	1570.39	1575.18	1578.97	1582.04	1586.47	1590.63	1603.94	1620.37	1634.67	1648.14	1659.09
12,000	1577.65	1582.84	1587.47	1591.53	1595.03	1599.58	1603.33	1606.14	1610.43	1614.60	1627.48	1643.52	1657.58	1670.64	1681.57
14,000	1603.07	1608.09	1612.50	1616.18	1619.74	1623.99	1627.74	1630.39	1634.65	1638.73	1651.06	1666.55	1680.29	1693.07	1703.78

TABLE IV

MEASURED SPEED OF SOUND IN SEA WATER, s = 35.02 ‰

P lbs/in^2 \ T°C	-2.970°	-1.846°	-0.986°	0.049°	0.944°	2.014°	3.060°	3.966°	5.083°	5.967°	9.937°	15.039°	20.298°	24.962°	29.834°
14.7	---	1440.60	1444.86	1449.73	1453.85	1458.50	1463.00	1466.82	1471.46	1474.95	1490.03	1507.15	1522.64	1534.59	1545.61
2,000	1457.61	1463.28	1467.37	1472.23	1476.34	1480.86	1485.42	1489.17	1493.84	1497.41	1512.47	1529.74	1545.35	1557.51	1568.79
4,000	1481.06	1486.52	1490.58	1495.35	1499.34	1504.02	1508.51	1512.17	1516.88	1520.35	1535.33	1552.76	1568.27	1580.35	1591.76
6,000	1505.04	1510.33	1514.34	1519.16	1523.03	1527.65	1532.06	1535.83	1540.27	1543.77	1558.57	1575.71	1591.06	1603.24	1614.58
8,000	1529.76	1534.86	1538.81	1543.47	1547.36	1551.67	1556.01	1559.67	1564.02	1567.45	1581.88	1598.72	1614.15	1625.92	1637.27
10,000	1554.60	1559.59	1563.41	1567.98	1571.86	1575.84	1580.22	1583.68	1588.10	1591.31	1605.30	1621.85	1636.99	1648.63	1659.80
12,000	1579.78	1584.69	1588.31	1592.74	1596.44	1600.24	1604.54	1607.91	1612.06	1615.34	1628.85	1644.91	1659.85	1671.12	1682.28
14,000	1605.00	1609.88	1613.16	1617.46	1621.12	1624.69	1628.97	1632.07	1636.15	1639.30	1652.37	1667.74	1682.56	1693.46	1704.52

TABLE V

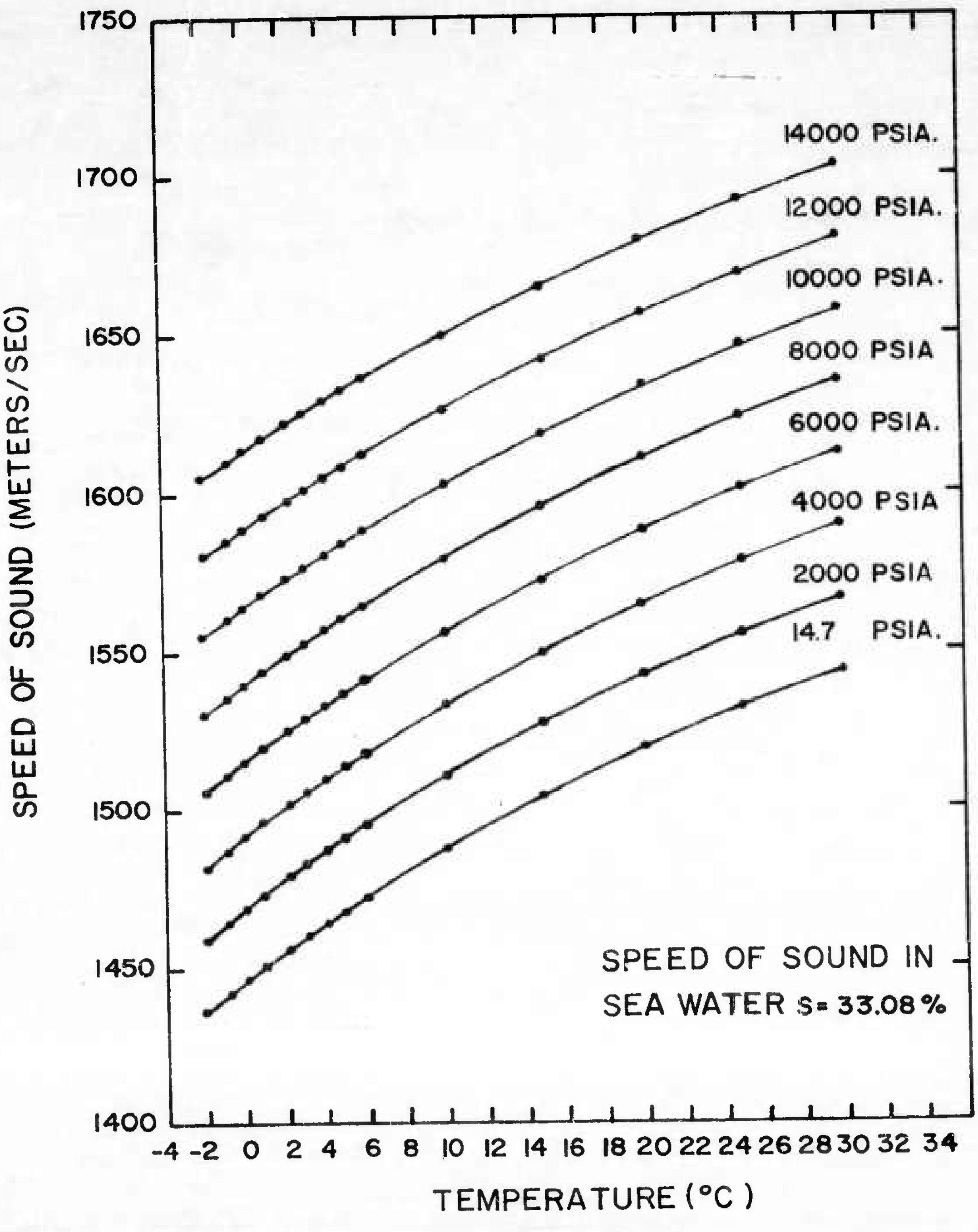
MEASURED SPEED OF SOUND IN SEA WATER, s = 36.02 ‰

T°C / P lbs/in²	-3.146°	-2.105°	-1.015°	-0.006°	1.031°	1.999°	2.987°	3.986°	4.989°	5.951°	10.038	14.936°	19.875°	24.770°	30.347°
14.7	---	1440.90	1446.09	1450.76	1455.42	1459.68	1463.99	1468.16	1472.47	1476.14	1491.42	1507.98	1522.56	1535.33	1547.54
2,000	1457.82	1463.50	1468.55	1473.09	1477.85	1482.08	1486.49	1490.56	1494.95	1498.80	1513.94	1530.75	1545.50	1558.38	1570.91
4,000	1481.33	1486.69	1491.64	1496.24	1501.01	1505.25	1509.49	1513.60	1517.92	1521.75	1536.88	1553.63	1568.37	1581.05	1593.92
6,000	1505.56	1510.54	1515.62	1520.08	1524.61	1528.73	1532.86	1537.21	1541.28	1545.09	1560.14	1576.69	1591.22	1603.92	1616.80
8,000	1529.98	1534.97	1539.78	1544.24	1548.81	1552.67	1556.79	1561.00	1565.09	1568.84	1583.48	1599.84	1614.03	1626.78	1639.46
10,000	1554.90	1559.68	1564.48	1568.83	1573.11	1577.06	1580.96	1584.92	1589.22	1592.69	1606.96	1623.07	1636.82	1649.45	1662.06
12,000	1579.97	1584.76	1589.22	1593.50	1597.63	1601.43	1605.26	1609.08	1613.09	1616.67	1630.42	1646.04	1659.54	1671.91	1684.48
14,000	1605.19	1609.97	1613.87	1617.94	1622.12	1625.84	1629.38	1633.13	1636.91	1640.58	1654.00	1669.04	1682.28	1694.47	1706.63

TABLE VI

MEASURED SPEED OF SOUND IN SEA WATER, s = 36.55‰

T°C / P lbs/in^2	-2.081°	-1.016°	0.009°	0.942°	2.043°	3.013°	3.954°	4.949°	5.996°	9.955°	14.909°	19.814°	24.903°	29.902°
14.7	1442.00	1447.04	1451.60	1455.80	1460.68	1464.92	1468.90	1473.02	1477.10	1492.13	1508.70	1523.24	1536.31	1547.34
2,000	1464.63	1469.63	1474.03	1478.34	1483.08	1487.36	1491.31	1495.44	1499.66	1514.68	1531.40	1545.92	1559.21	1570.43
4,000	1487.72	1492.85	1497.14	1501.36	1506.18	1510.33	1514.36	1518.31	1522.70	1537.47	1554.17	1568.79	1582.10	1593.36
6,000	1511.52	1516.51	1521.03	1525.05	1529.88	1533.89	1537.72	1541.87	1546.01	1560.66	1577.17	1591.62	1604.87	1616.17
8,000	1536.12	1540.92	1545.28	1549.30	1554.07	1557.90	1561.62	1565.66	1569.74	1584.11	1600.27	1614.53	1627.74	1639.03
10,000	1560.99	1565.67	1569.78	1573.71	1578.35	1582.14	1585.67	1589.64	1593.60	1607.76	1623.37	1637.49	1650.51	1661.62
12,000	1585.98	1590.59	1594.38	1598.37	1602.80	1606.42	1609.85	1613.72	1617.55	1631.28	1646.61	1660.38	1672.99	1684.07
14,000	1611.06	1615.43	1619.11	1623.02	1627.30	1630.66	1634.10	1637.70	1641.51	1654.99	1669.81	1683.11	1695.44	1706.28



1750
1700
1650
1600
1550
1500
1450
1400
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34
SPEED OF SOUND (METERS/SEC)
TEMPERATURE (°C)
14000 PSIA.
12000 PSIA.
10000 PSIA.
8000 PSIA
6000 PSIA.
4000 PSIA
2000 PSIA
14.7 PSIA.
SPEED OF SOUND IN
SEA WATER S= 33.08%

FIG. 7

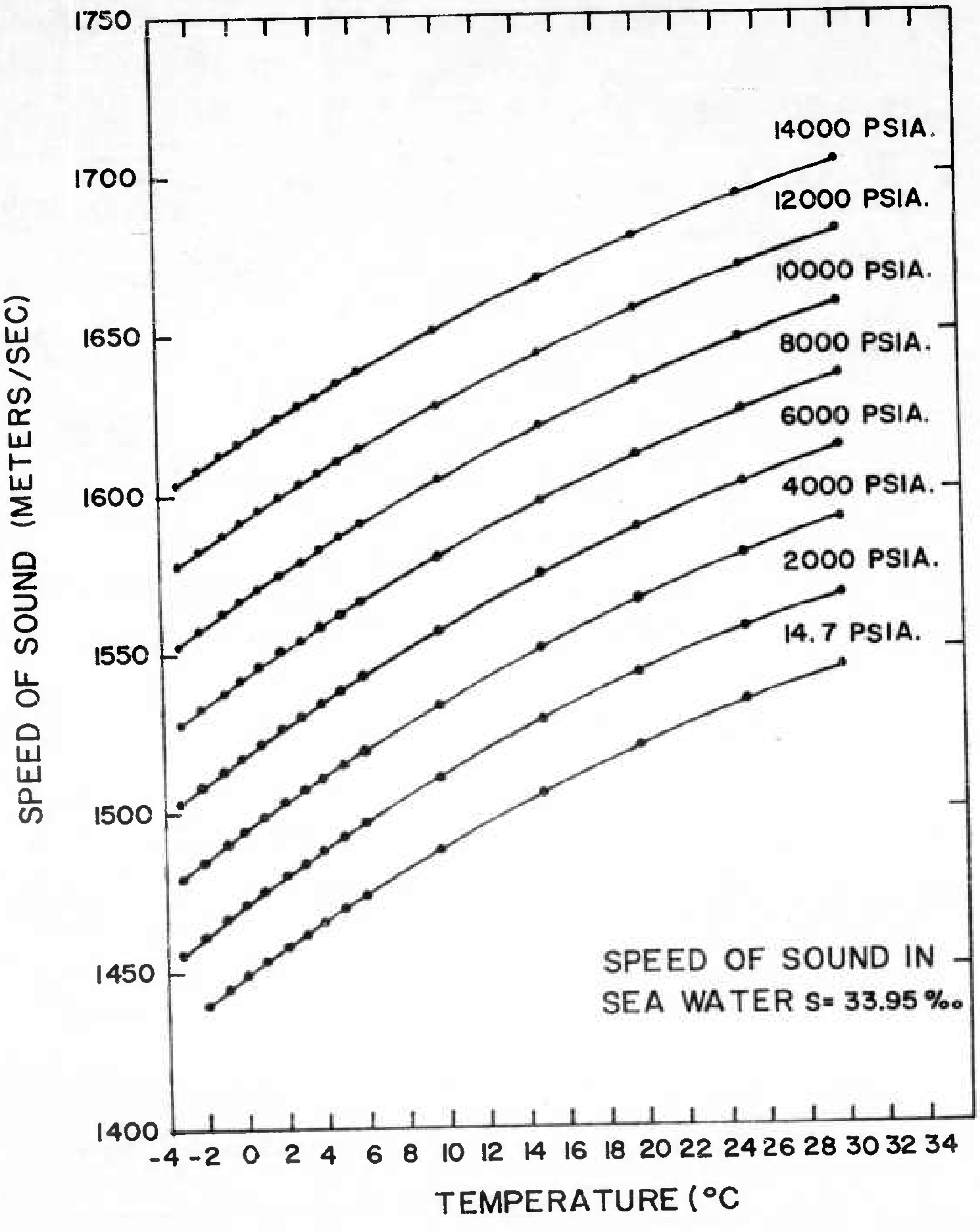

1750
1700
1650
1600
1550
1500
1450
1400
SPEED OF SOUND (METERS/SEC)
14000 PSIA.
12000 PSIA.
10000 PSIA.
8000 PSIA.
6000 PSIA.
4000 PSIA.
2000 PSIA.
14.7 PSIA.
SPEED OF SOUND IN
SEA WATER S= 33.95 ‰
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34
TEMPERATURE (°C

FIG. 8

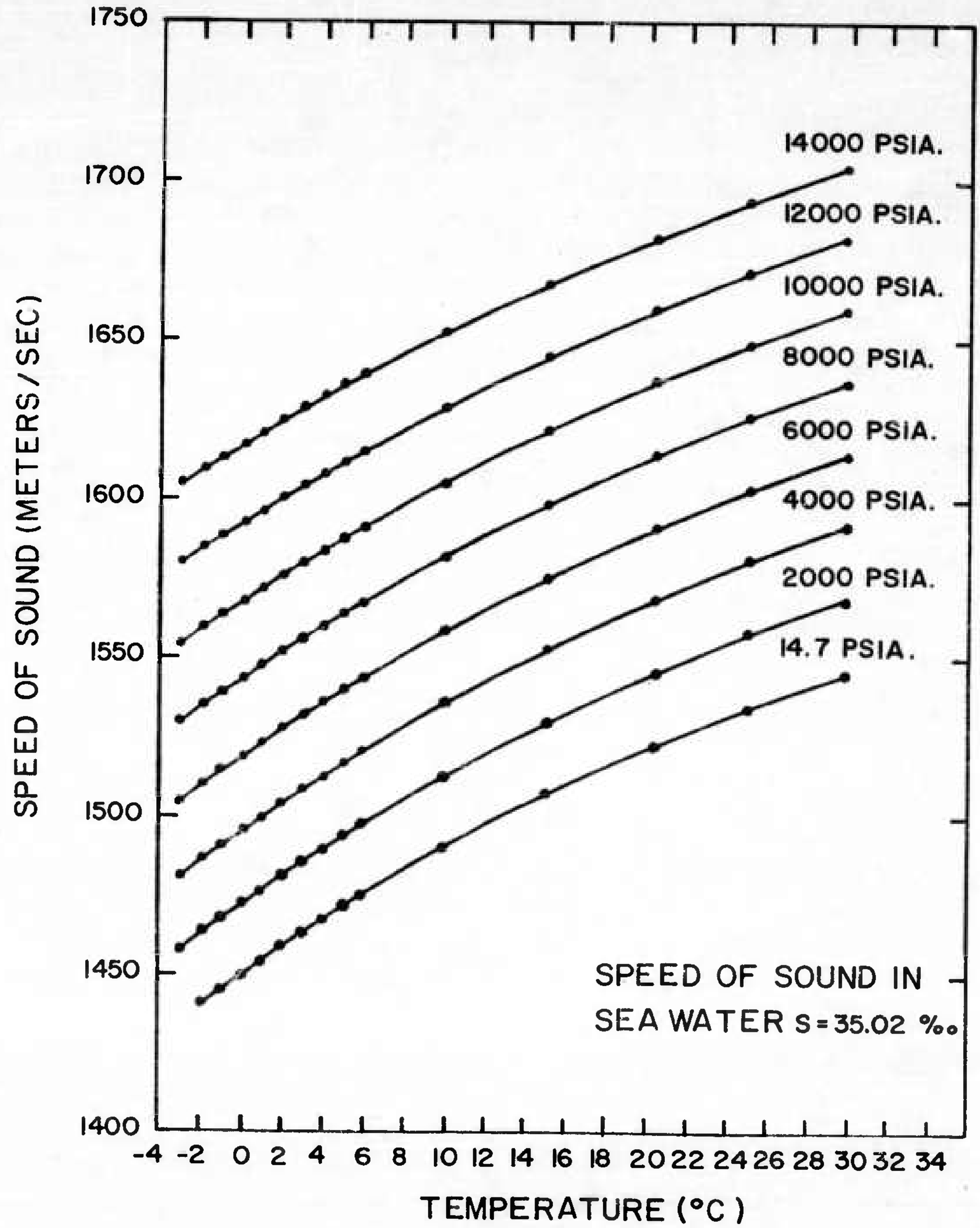
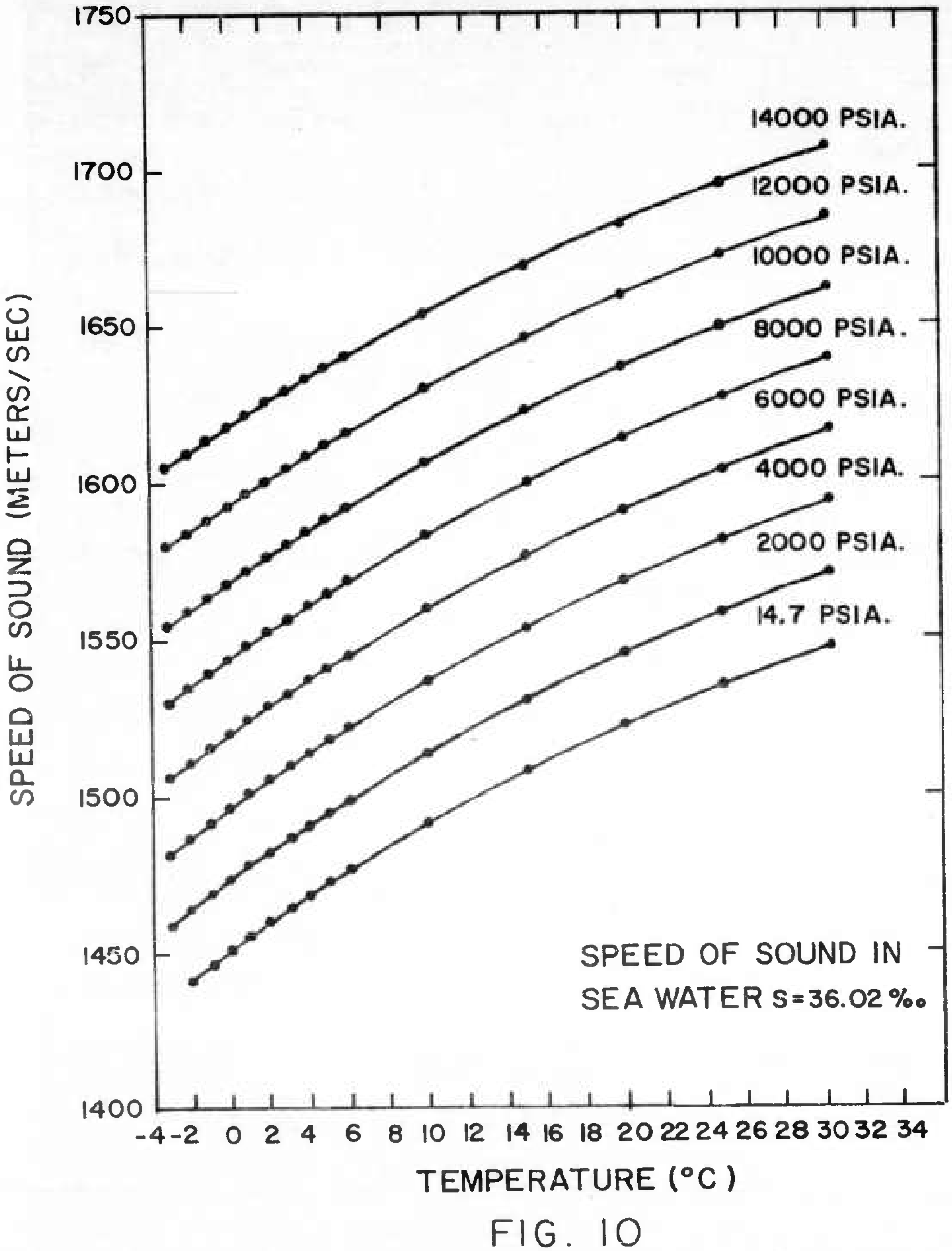

SPEED OF SOUND (METERS/SEC)
1750
1700
1650
1600
1550
1500
1450
1400
14000 PSIA.
12000 PSIA.
10000 PSIA.
8000 PSIA.
6000 PSIA.
4000 PSIA.
2000 PSIA.
14.7 PSIA.
SPEED OF SOUND IN
SEA WATER S = 35.02 ‰
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34
TEMPERATURE (°C)

FIG. 9



1750
1700
1650
1600
1550
1500
1450
1400
SPEED OF SOUND (METERS/SEC)
14000 PSIA.
12000 PSIA.
10000 PSIA.
8000 PSIA.
6000 PSIA.
4000 PSIA.
2000 PSIA.
14.7 PSIA.
SPEED OF SOUND IN
SEA WATER S=36.02 ‰
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34
TEMPERATURE (°C)

FIG. 10

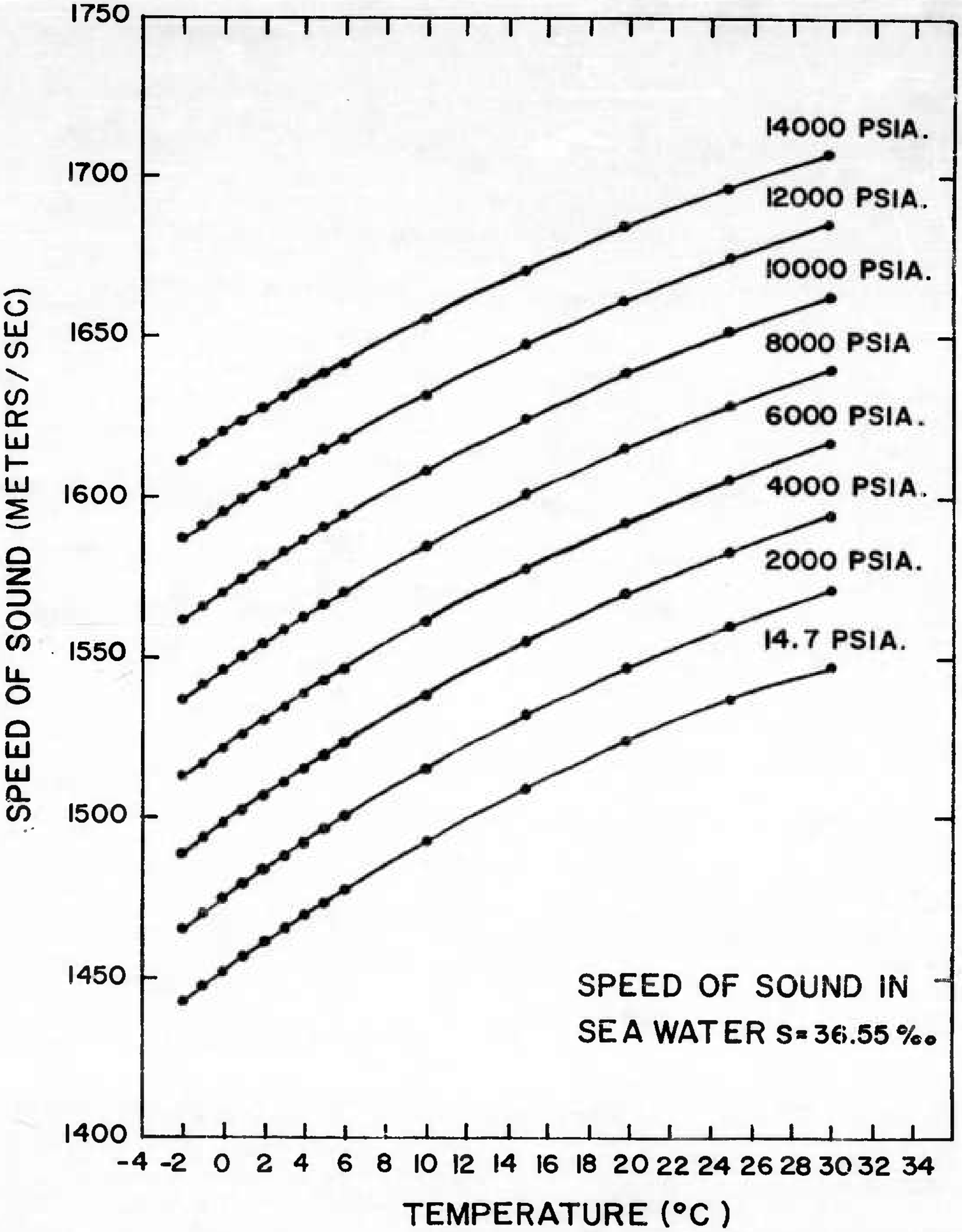

1750
1700
1650
1600
1550
1500
1450
1400
14000 PSIA.
12000 PSIA.
10000 PSIA.
8000 PSIA
6000 PSIA.
4000 PSIA.
2000 PSIA.
14.7 PSIA.
SPEED OF SOUND (METERS / SEC)
SPEED OF SOUND IN
SEA WATER S=36.55 ‰
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34
TEMPERATURE (°C)

FIG. 11

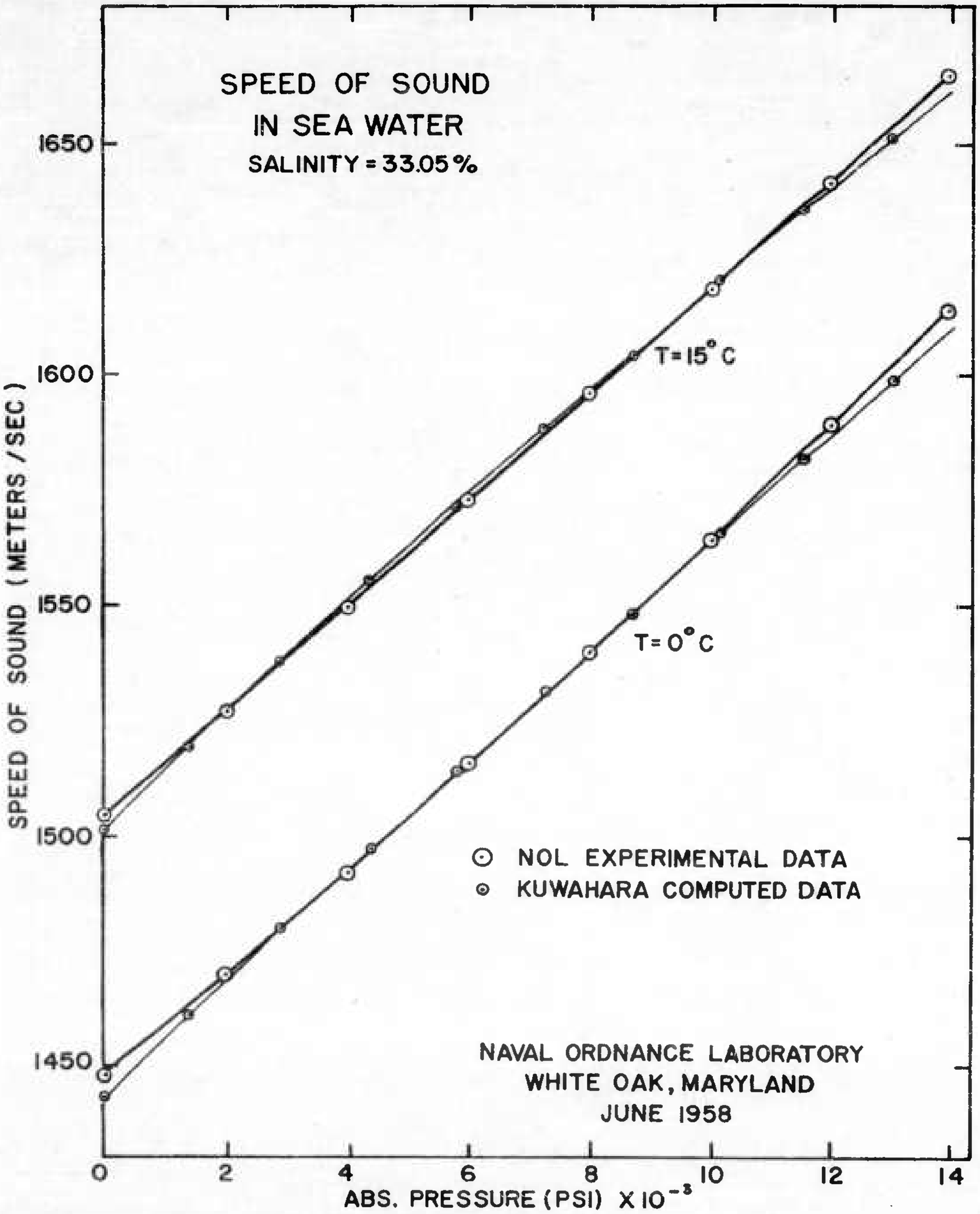

SPEED OF SOUND
IN SEA WATER
SALINITY = 33.05%
1650
1600
1550
1500
1450
SPEED OF SOUND (METERS / SEC)
T = 15° C
T = 0° C
NOL EXPERIMENTAL DATA
KUWAHARA COMPUTED DATA
NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND
JUNE 1958
0
2
4
6
8
10
12
14
ABS. PRESSURE (PSI) X 10⁻³

FIG. 12

17. The differences between the measured sound speeds and the predicted sound speeds at atmospheric pressure were discussed by V. Del Grosso[13]. To account for the differences between the computed and measured sound speeds at other pressures a review was made of the method used for computing sound speeds from specific volume data. Kuwahara and Matthews computed sound speeds in sea water from Newton's formula

$$c^2 = \frac{\gamma}{\rho\beta} , \tag{1}$$

where γ is the ratio of specific heats, ρ is the density, and β is the isothermal compressibility. The density was computed from the formula

$$\rho = \frac{\rho_0}{1 - \mu} \tag{2}$$

where μ is the mean compressibility per bar between the pressures $P_0 = 0$ and $P = P$. The true compressibility, β, was found from the mean compressibility by the relation,

$$\beta = \frac{\mu + P \frac{d\mu}{dP}}{1 - P\mu} . \tag{3}$$

It may be seen from this equation that the mean compressibility is defined by

$$\mu = -\frac{1}{V_0} \frac{(V - V_0)}{(P - P_0)} \tag{4}$$

where $P_0 = 0$. Substituting Eqs. (2) and (3) into Eq. (1), we obtain;

$$c^2 = \frac{\gamma(1 - P\mu)}{\rho_0(\mu + P \frac{d\mu}{dP})} . \tag{5}$$

The object in writing the equation for sound speed in terms of μ instead of β was to allow the use of an empirical equation obtained by V. Ekman[14] for μ. Ekman's equation for μ is written as a function of temperature, pressure, and salinity. This empirical equation was based principally on Amagat's[15] specific volume tables for distilled water and a few representative measurements of the specific volume of sea water made by Ekman. The pressure dependence of μ in Ekman's equation relies upon Amagat's pressure measurements. Therefore the effect of pressure on the speed of sound in water should have the same

characteristics regardless of whether Ekman's or Amagat's data is used. In Eq. (5) μ can be obtained either from Eq. (4) and Amagat's data, or from Ekman's empirical equation. If Eq. (4) and Amagat's data are used the second derivatives of sound velocity in distilled water with respect to pressure are positive. If Ekman's equation for μ is used the second derivatives are negative. It is apparent, therefore, that the negative curvature of sound speed as a function of pressure have been introduced by the inclusion of Ekman's work. Since Ekman's equation is estimated to be accurate to about three parts per thousand[16], sound speeds computed from Ekman's equation cannot be more accurate than 4.5 m/sec. In actuality, the differences observed between the measured and the computed sound speeds are less than this amount. Comparison of the effect of pressure on the speed of sound is not possible in other work[2,3,4] because of the large pressure increments taken and the small amount of data obtained in the pressure range considered here.

PRECISION AND ACCURACY OF THE INSTRUMENTATION

18. In this report precision has to do with the accidental or random errors. On the other hand, the accuracy pertains to the systematic errors associated with the measurements in the absence of random errors. The precision of the measurements is obtained by fitting equations[9] to the measured data by the method of least squares. If the equation is a good approximation to the mean of the experimental data, residuals obtained from the differences between the computed and the measured sound speeds may be used to obtain a standard deviation. The standard deviation from the mean is then taken as a measure of the precision of the experimental data.

19. Proceeding in this manner, the algebraic sum of the differences between the computed data and the experimental data divided by the number of measurements (i.e., the average error) was 0.00 m/sec and 0.01 m/sec for distilled water and sea water, respectively. It is concluded therefore that the equations[9] obtained to fit the experimental data are good approximations to the mean of the experimental data. The standard deviation of the experimental data from the mean, or, the precision, was 0.17 m/sec in distilled water and 0.22 m/sec in sea water. The precision of the data is about one part in 7500.

20. The systematic errors associated with the sound speed measurement using the velocimeter are caused mainly by absorption, dispersion, viscosity, pressure differential across the

bellows, and the effects of crystal distortion caused by this pressure difference. A review of the magnitude of the above errors and other possible sources of error has been made and is summarized in Table VII. At atmospheric pressure the contributions of the pressure differential and the crystal distortion are zero; the systematic error, found by adding the individual errors algebraically, is consequently +0.03 m/sec. At a pressure of 14,000 psi, the errors add to give +0.01 m/sec. In Table VII the sign of the individual error was assigned to correct the measured sound speed to correspond to the sound speed for an unbounded medium. Consequently, the estimated maximum systematic error, +0.03 m/sec, indicates that the measured sound speed is less than the sound speed in an infinite medium by 0.03 m/sec.

SUMMARY

21. It may be concluded that the instrument available at NOL for the measurement of sound speeds in water as a function of temperature and pressure can achieve a precision of 1 part in 7,500 under normal use. If measurements in other liquids are considered the precision with which the sound speeds may be obtained will depend upon the variation of sound speed with temperature and pressure in that particular liquid. In dispersive liquids, corrections to the speed of sound may have to be made to account for the ultrasonic frequencies used in this instrument.

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TABLE VII

SYSTEMATIC ERRORS

Type of Error	Magnitude of Error (Meter/sec)
Pressure differential*	+0.010
Crystal deflection*	-0.030
Pulse amplitude	-0.002
Shear viscosity (at tube walls)	+0.029
Bulk viscosity	0.000
Radial heat conduction	0.000
Heat conduction between compression and rarefactions	0.000
Molecular scattering	0.000
Molecular and chemical absorption	0.000
Change in frequency due to absorption	0.000
Change in velocity due to dispersion	+0.003
Time delay during reflection	0.000

*These errors go to zero as the pressure approaches atmospheric pressure.

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